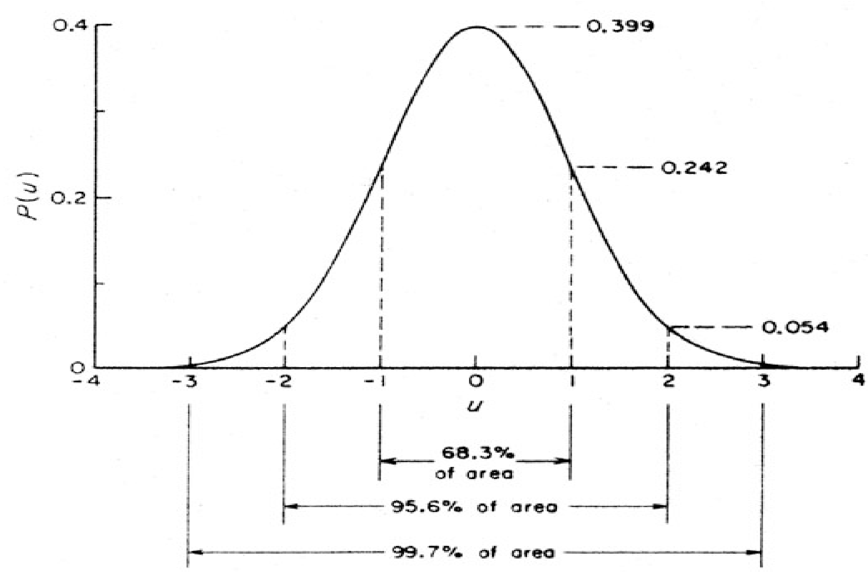
The Normal Probability Distribution



Properties of the normal distribution:

1- Symmetric: Mean = Median.

2- Continuous for all possible values of x between $-\infty$ and $+\infty$ so that any interval of real numbers has a probability greater than zero.

3- $-\infty \leq x \leq +\infty$.

4- Only the first two moments (i.e., mean (μ) and standard deviation (σ)) are needed to characterize the normally distributed variable. Note that the shape of the normal is not unique since the shape is determined by μ and σ.

5- The formula of the normal probability density function is given by,

$$f(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

The value of the function indicates the probabilty of observing a particular deviation from the mean determined by the specified number of standard deviations.

The value of this function is maximized at the zero deviation (i.e., $x = \mu$). Thus, the average is the most expected value of *x*. Remember that the expectations are taken unconditionally as only the observed values of *x* are needed to calculate the average.

6- 68.26% of all observations fall within one standard deviation, 95.44% of all observations fall within two standard deviations, and 99.74% of all observations fall within three standard deviations.

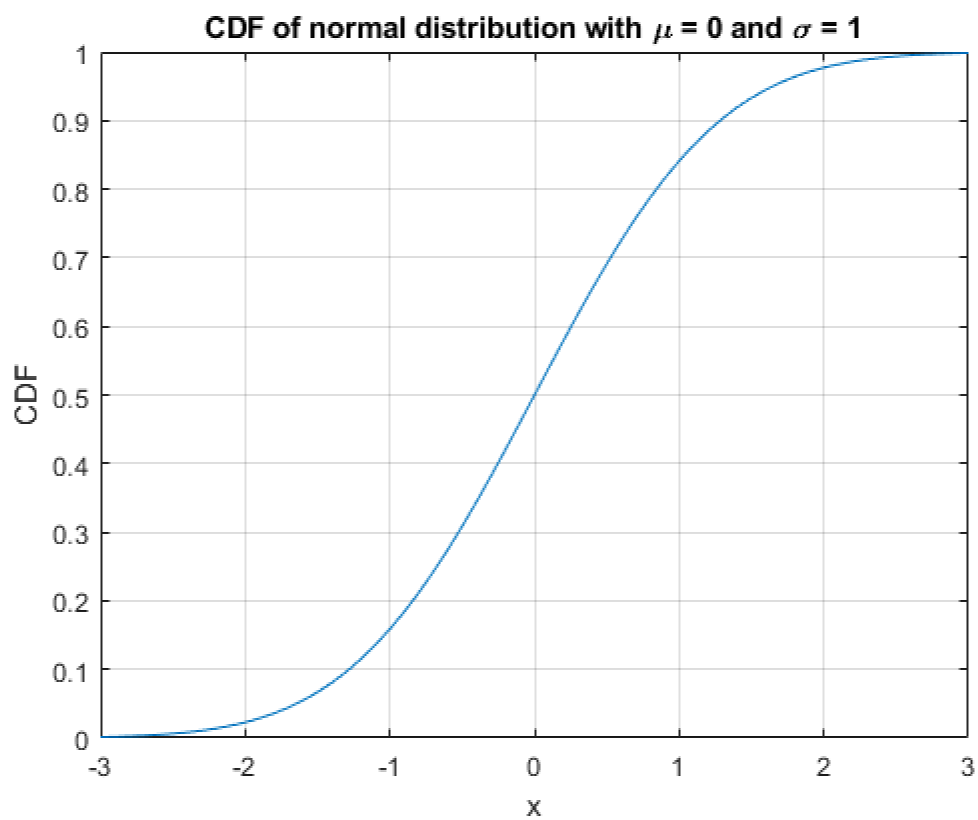
7- Stable under addition: combing two normally distributed variables results in normally distributed variable.

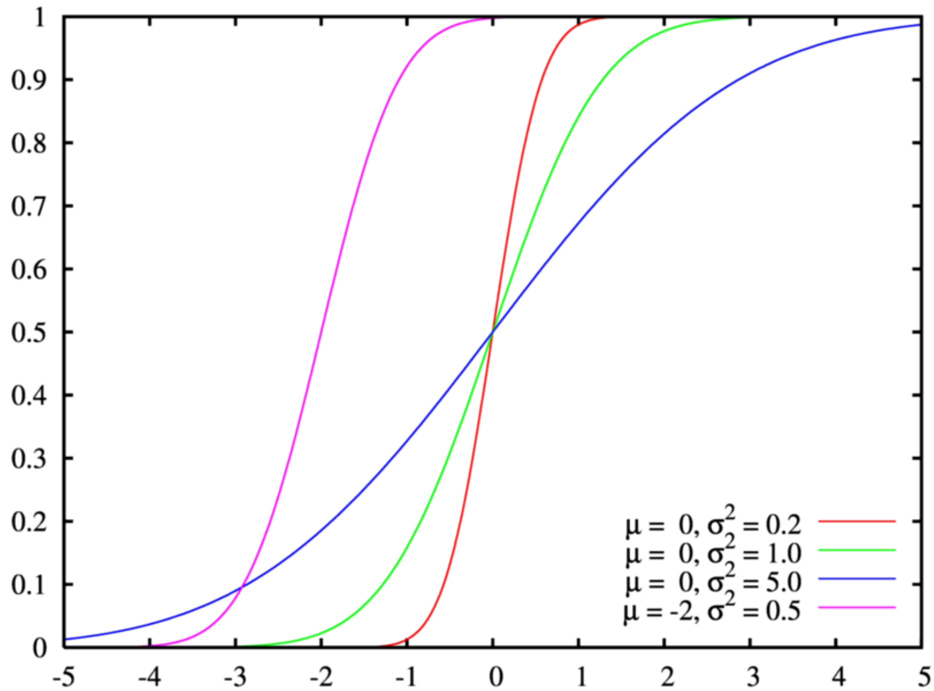
8- Linearity in parameters: the impact of the variable that is believed to affect the normally distributed variable is linear. This does not necessarily mean that the relationship between the two variables will be linear. Thus, the relationship between the two variables can be written in the form:

$$y_i = \alpha + \beta x_i + e_i$$

9- The cumulative density function of the normally distributed variable is given by,

$$F(x) = \int_{-\infty}^{x} f(x)\ dx$$




1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-5
-4
-3
-2
-1
0
1
2
3
4
5
μ = 0, σ² = 0.2
μ = 0, σ² = 1.0
μ = 0, σ² = 5.0
μ = -2, σ² = 0.5